Evaluating the Effects of Investment Decisions on Risk-Adjusted Returns

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Decomposition

- Finance's standard approach to ex-post evaluation of causation:
  - If there are 2 causes of 7, then they must have values 5 & 2 since 7 = 5+2.
- Leads to problems.
- Examples:
  - Performance Attribution (Multi Decision Layered)
  - Risk Attribution (Menchero&Hu)(Bertrand)
  - Also in Trade Returns and Contributions

Decomposition
= Top-Down modeling

1. Portfolio-level (Total)

   Impose economic requirements

   that completely specify.

2. Component-level (Issue or Decision)

   Rely upon arbitrary assumptions
   & then Partially restrict them.

Decomposing Active Return for Multiple Kinds, k, of Decisions for Attribution

$\Delta R \equiv R^P - R^B = \sim = All + Sel + Int \equiv \Sigma_k X_k$

Impose Portfolio-level Definition of ΔR

And formal requirement on sum of attributes

And, perhaps, intuitive monotonicity of

- All w/over-weighting good sectors and
- Sel w/over-weighting good issues in sectors

Decomposing Active Return with freedom

$\Delta R \equiv R^P - R^B = All' + Sel' + Int' \equiv \Sigma_k X'_k$

$= (All + \mathbf{Ae}) + (Sel + \mathbf{Se}) + (Int + \mathbf{Ie})$

where Ae + Se + Ie = 0. Thus, $\mathbf{X_k}$ **arbitrary!**

Can even still save intuitive monotonicity of

- All' w/over-weighting good sectors and
- Sel' w/over-weighting good issues in sectors

By fully imposing Meaning
Only at the Portfolio level,
& then restrictions below,
We do <u>not</u> get valid meaning
at the component (attribute) level.

Decomposing Risk-Adjusted Returns

$$IR \equiv \frac{<\Delta R>}{\sigma(\Delta R)} = \frac{\Sigma_k <X_k>}{\sigma(\Delta R)}$$

$$= \Sigma_k \frac{[\rho(X_k, \Delta R) * \sigma(X_k)]}{\sigma(\Delta R)} * \frac{<X_k>}{[\rho(X_k, \Delta R) * \sigma(X_k)]}.$$

$$\equiv \Sigma_k WR_k * IR_K$$

where $\rho \equiv$ Correlation $\rightarrow \Sigma_k WR_k = 1.$

Decomposing Risk-Adjusted Returns

- $IR \equiv \dfrac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \dfrac{\Sigma_k \langle X_k \rangle}{\sigma(\Delta R)}$

$$= \Sigma_k \frac{[\rho(X_k,\Delta R) * \sigma(X_k) + E_k]}{\sigma(\Delta R)} * \frac{\langle X_k \rangle}{[\rho(X_k,\Delta R) * \sigma(X_k) + E_k]}.$$

$$\equiv \Sigma_k WR'_k * IR'_K.$$

$\Sigma_k E_k = 0 \rightarrow \Sigma_k WR'_k = 1.$

But, since E_k is arbitrary, **IR'_K is Arbitrary!**

Decomposing Risk-Adjusted Returns

$$IR \equiv \frac{\langle \Delta R \rangle}{\sigma(\Delta R)} = \frac{\Sigma_k \langle X_k \rangle}{\sigma(\Delta R)} = \frac{\Sigma_k [\langle X_k \rangle + aa_k]}{\sigma(\Delta R)}$$

$\equiv \Sigma_k IRCont_k$, where $\Sigma_k aa_k = 0$.

$IRCont_k \equiv$ "The portion of IR attributed to decision k."

But $\mathbf{IRCont_K}$ **is Arbitrary!**

<u>Note</u>: All the decompositions of IR have the same denominator, $\sigma(\Delta R)$.

Again,
By fully imposing Meaning
<u>Only</u> at the Portfolio level,
We do <u>not</u> get valid meaning
at the component (attribute) level.

Properties
(like returns and attributes)
need to be
economically defined
<u>at the level they are employed</u>,
else the arbitrariness breeds
nonsense.

HATS

Top-Down Modeling

The Decomposition of the ‘Hat Quantity’

What is the ‘cause’ of there being 14 hats between us?

HAT Number: #H_j

- HATS: You 11H & Me 3H
- Model: 14H = 9H + 5H
- Top-level meaning:
  - 1. Correctly Adds up: 9 + 5 = 14
- Lower level, only formal:
  - 2. Correctly ordered: Y>M.
  - 3. Correct Parity: All odd.
  - 4. Correct Units: H.
- Still Wrong since not defined at atomic level:#H_j=?
- Decomposed Total H → Incorrectly model #H_j.

Decision Evaluation

- Attribution to **<u>Controllables</u>**:
  Evaluate the effects of decisions,
  as opposed to the effects of the market.
- Avoid arbitrariness
  Define these effects

<u>at the level of individual decisions</u>.

Will here apply decision evaluation
to Risk Attribution.

Decision Process

- Choose investment process employed.
- EXAMPLE w/Simple Single-Period (multiple sub-periods):

- Null: $R^B = \Sigma_S W^B{}_S \; R^B{}_S$
- Allocation: $R^A = \Sigma_S W^P{}_S R^B{}_S$
  $= \Sigma_S W^P{}_S \Sigma_{j\varepsilon S} W^B{}_j R_j$
- Selection: $R^P = \Sigma_S W^P{}_S R^P{}_S$
  $= \Sigma_S W^P{}_S \Sigma_{j\varepsilon S} W^P{}_j R_j$

- For now, Forgo the harder:
  - Compounded Multi-Period,
  - Nested
  - Forking Tree

DEFINE ‘ATTRIBUTE’

The value of a controllable decision level attribute

≡ The effect on the parameter

of implementing the decision

within a set of multi-layered decisions ≡ Difference =

The value of the parameter (e.g. IR)

after implementing the decision

Minus

- The value of the parameter

before implementing the decision.

This def. vs. Standard PA for FI & multi: T, Curr, Asset, Nesting.

Decision Process

- EXAMPLE w/ days:
- Null: $R^B = \Sigma_S W^B_S \; R^B_S$
- Allocation: $R^A = \Sigma_S W^P_S \; R^B_S$
- Selection: $R^P = \Sigma_S W^P_S R^P_S$

Decision Evaluation Ex.

- Chose Information Ratio
- $IR^{P,B} \equiv \dfrac{<\Delta R>}{\sigma(\Delta R)} = \dfrac{<R^P - R^B>}{\sigma(R^P - R^B)}$

- Allocation: $IR^{All} \equiv IR^{A,B} - IR^{B,B} = IR^{A,B}$
- Selection: $IR^{Sel} \equiv IR^{P,B} - IR^{A,B} = IR^{P,B} - IR^{A,B}$

Total: $IR^{P,B}$

Decision Risk

The decision

To select holdings within sectors

On each of the days in the period

Increased the portfolio's IR by

$$IR^{Sel} = IR - IR^{A,B} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)}$$

= The amount of IR attributed to Selection.

Decision Evaluation V. Decomposition Cont.

$$IR^{Sel} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^A - R^B)} = \text{Effect of Sel on IR.}$$

Answers the precise Economic question re difference:

How much did the implementation of ‘Sel’ decision increase the information ratio?

Whereas Decompostion’s

$$IR\_Cont_A = \frac{\langle R^P - R^A \rangle}{\sigma(R^P - R^B)} = \frac{\langle R^P - R^B \rangle}{\sigma(R^P - R^B)} - \frac{\langle R^A - R^B \rangle}{\sigma(R^{\mathbf{P}} - R^B)}$$

Is an economically ill-formed “measure”

Constructed by an arbitrary formal process and then Named “Contribution of Selection to IR.”

Misinformation from Standard Decomposition

If R^P tracked R^B

But R^A did not track R^B at all,

Then can have cases where

Standard Decomposition results imply that

  Selection does not contribute significantly to IR

When in fact, it contributes a lot.

And Decision Evaluation precisely shows this.

IR Attribution Example

- Consider,

  a BM, B,

  a state of the Fund after Allocation, A,

  and a final position of the Fund, P. $e \ll 1$

- The returns over 4 periods are:

B:	-1/2	1/2	1/2	-1/2
A	3/2	1/2	1/2	3/2
P	3/2+e	5/2-e	5/2-e	3/2+e

IR Attribution Ex. Results

A-B	2	0	0	2
	<A-B>=1 & SD(A-B) = 1			
P-B	2+e	2-e	2-e	2+e
	<P-B>=2 & SD(P-B) = e			

- **Decomposition**: e=0.1

Allocation = <A-B>/SD(P-B) =1/e Medium 10

Selection = <P-B>/SD(P-B) - <A-B>/SD(P-B) = 2/e –1/e Medium 10

- **FDE**:

Allocation = <A-B>/SD(A-B) = 1/1 = 1 Small 1

Selection = <P-B>/SD(P-B) - <A-B>/SD(A-B) = 2/e –1/1 Large 19

IRR Attribution Ex's Conclusion

- Allocation → fund w/ gain & wild fluctuations.
- Selection → fund w/ same additional gain
  & decreased fluctuations.

- Thus, Allocation → small IR,
- While Selection → large IR.

- Decomposition → wrong.
- **FDE → exactly right**.

Summary

- Standard methods of Decomposition
  - Define at the portfolio level,
    Interpret at the decision or issue level.
  - Leading to formally consistent but misleading results.
- Decision Evaluation
  - Provides uniquely appropriate answers
    to precisely meaningful economic questions
    at the level sought.

Warning!

Standard Decomposition Methods
can Dangerously Misguide

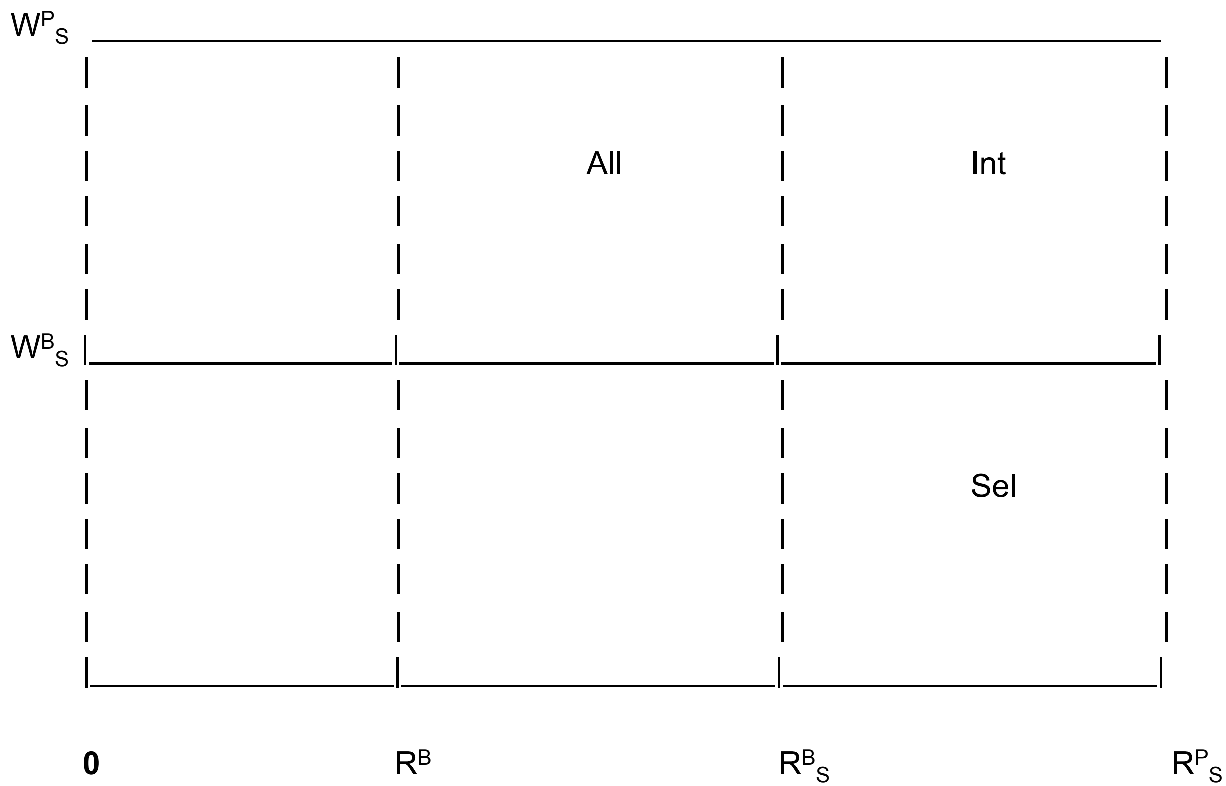
any Investment Processed that relies on knowing:

- Performance Attribution of Brinson-type, or
- Risk Attribution &Contribution
- OTHER SO DERIVED COMMON PARAMETERS,
  - Like component-level Trade inclusive returns & Contributions

Use Economically Meaningful Methods.

Decision Evaluation→Meaningful Results.

Attribute Definitions



Reference paper

http://www.northinfo.com/documents/377.pdf