

Performance attribution for multi-layered investment decisions

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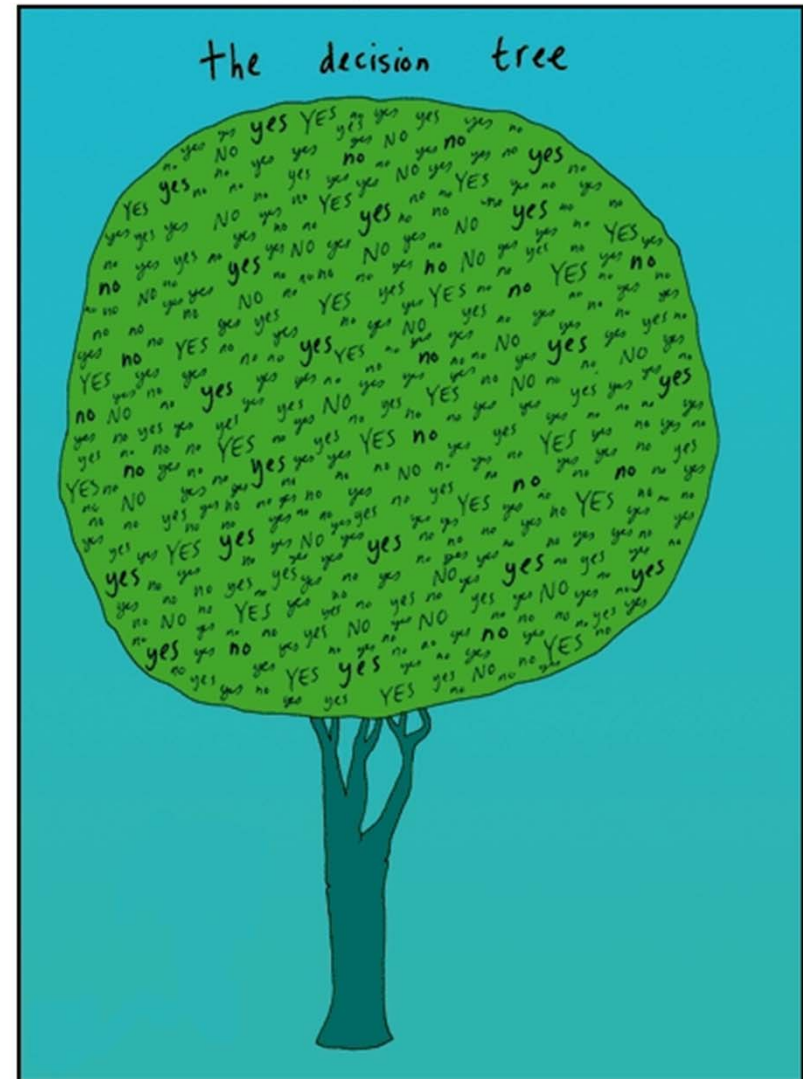
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The attribution problem

- Often investments involve multiple layers of decisions
- For example, in asset allocation the weight in each asset class is usually a result of several decisions
- For instance, the allocation to US small cap value equity depends on how much we allocate to:
 - equity vs fixed income,
 - US equity vs international equity,
 - large cap vs small cap, and
 - value vs growth
- The goal is to measure the impact of each allocation decision

Solution

- The proposed approach disentangles layers of investment decisions and measures impact of each decision separately
- It is most suited for performance attribution in asset allocation but can also be used in other areas such equity and fixed income
- The algorithm is very simple and can be easily implemented for arbitrary decision trees





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Algorithm

Total active return by asset groups

Total active return of a portfolio is equal to:

$$R_{Portfolio} - R_{Benchmark} = \sum_i w_i R_i - \sum_i w'_i R_i = \sum_i R_i (w_i - w'_i) \quad (1)$$

where w_i = portfolio weight of asset i

w'_i = benchmark weight of asset i

R_i = return of asset i

It can be divided into active returns of major groups of assets. For example, active return of equity (group G_1) and fixed income (group G_2):

$$(1) = \sum_{i \in G_1} R_i (w_i - w'_i) + \sum_{i \in G_2} R_i (w_i - w'_i) \quad (2)$$

The key formulas

Next, let's break down the active return of each group into two components:

$$\sum_{i \in G_k} R_i (w_i - w'_i)$$

Active return
of the group

$$(3) \quad \sum_{i \in G_k} R_i \left(w_i - w'_i \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i} \right) + \sum_{i \in G_k} R_i w'_i \left(\frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i} - 1 \right) \quad (4)$$

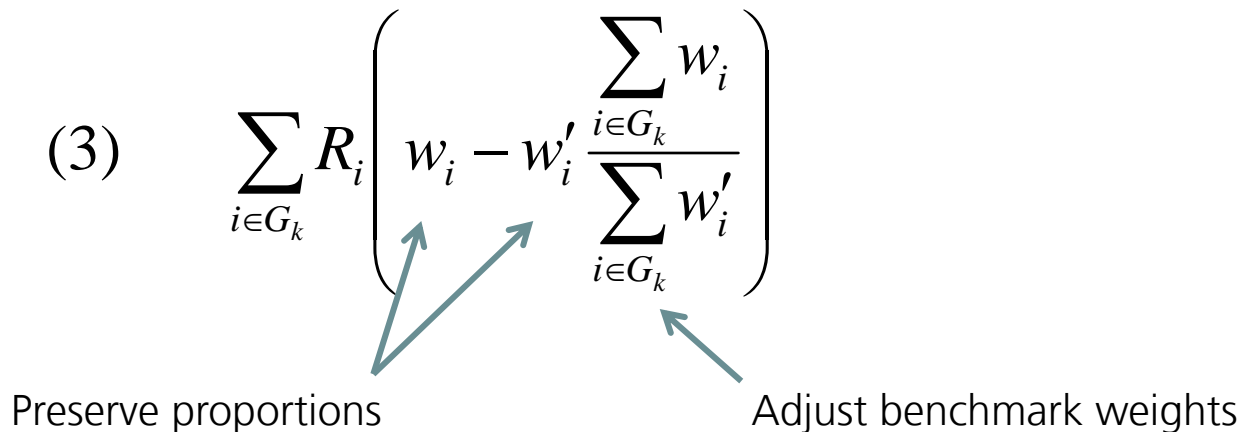
Impact of decisions
within the group

Impact of under/overweight
of the group itself

Impact of decisions within the group

- In (3), we adjust benchmark weights by ratio: $\frac{\text{portfolio weight of the group}}{\text{benchmark weight of the group}}$
 - In effect, we adjust the benchmark weight of the group to the portfolio level
 - This removes the impact of the group under/overweight
 - Meanwhile we preserve the proportions of the weights within the group for both the portfolio and the benchmark
 - This allows us to measure the impact of decisions within the group

$$(3) \quad \sum_{i \in G_k} R_i \left(w_i - w'_i \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i} \right)$$



Preserve proportions

Adjust benchmark weights

Impact of under/overweight of the group itself

- In (4), we use the benchmark return of the group and multiply it by the group under/overweight
 - Thus, we use the benchmark composition of the group and
 - Measure the pure impact of the group under/overweight

$$\sum_{i \in G_k} R_i w'_i \left(\frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i} - 1 \right) \quad (4)$$

Use benchmark composition

Multiply by group under/overweight

Combining group impacts

To continue with the example of two groups, let's combine the impacts of group under/overweights to measure impact of equity vs fixed income decision

$$\begin{aligned}
 (2) = & \sum_{i \in G_1} R_i \left(w_i - w'_i \frac{\sum_{i \in G_1} w_i}{\sum_{i \in G_1} w'_i} \right) + \sum_{i \in G_1} R_i w'_i \left(\frac{\sum_{i \in G_1} w_i}{\sum_{i \in G_1} w'_i} - 1 \right) + \\
 & + \sum_{i \in G_2} R_i \left(w_i - w'_i \frac{\sum_{i \in G_2} w_i}{\sum_{i \in G_2} w'_i} \right) + \sum_{i \in G_2} R_i w'_i \left(\frac{\sum_{i \in G_2} w_i}{\sum_{i \in G_2} w'_i} - 1 \right)
 \end{aligned}$$

Impact of equity vs
fixed income decision

Further steps

- We leave the second component of each group (impact of under/overweight of the group itself) as is
- This will be the final component of the attribution
- We continue dividing the first component of each group (impact of decisions within the group) in the same way as we divided the total active return of the portfolio

Dividing into subgroups

Denote $w'_i \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i}$ by w''_i , then the first component is equal to:

$$(3) = \sum_{i \in G_k} R_i \left(w_i - w'_i \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w'_i} \right) = \sum_{i \in G_k} R_i (w_i - w''_i)$$

which is the same as formula (1) and it can be similarly divided into subgroups and decomposed into formulas (3) and (4)

Reaching the most granular level

We keep dividing groups into finer and finer subgroups until we reach the most granular level, i.e. when a group consists of a single asset

At the most granular level, the first component (impact of decisions within the group – formula (3)) is equal to zero:

$$\sum_{i \in G_k} R_i \left(w_i - w_i^{\text{norm}} \frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i^{\text{norm}}} \right) = R_i \left(w_i - w_i^{\text{norm}} \frac{w_i}{w_i^{\text{norm}}} \right) = 0$$



And the entire active return of the group (single asset) is equal to the second component (impact of its under/overweight – formula (4))

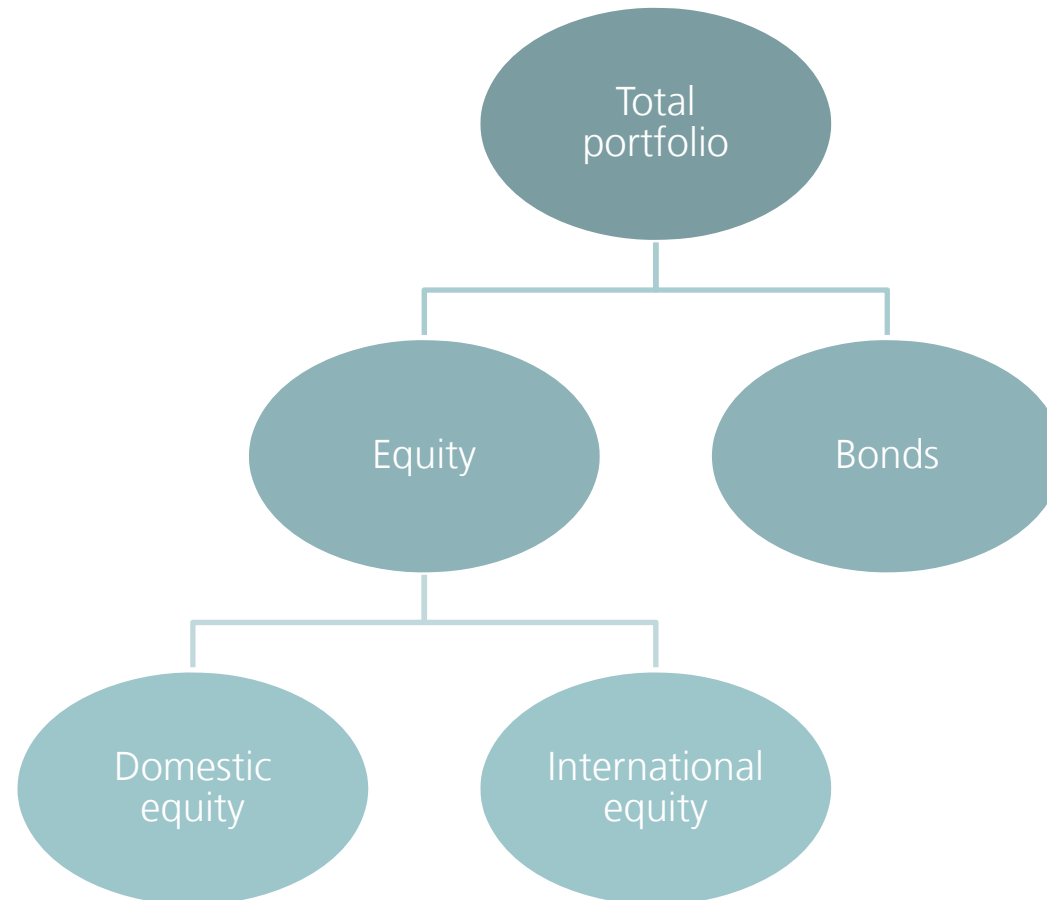
$$\sum_{i \in G_k} R_i w_i^{\text{norm}} \left(\frac{\sum_{i \in G_k} w_i}{\sum_{i \in G_k} w_i^{\text{norm}}} - 1 \right) = R_i w_i^{\text{norm}} \left(\frac{w_i}{w_i^{\text{norm}}} - 1 \right) = R_i (w_i - w_i^{\text{norm}})$$



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Three asset example

Multiple layers of investment decisions

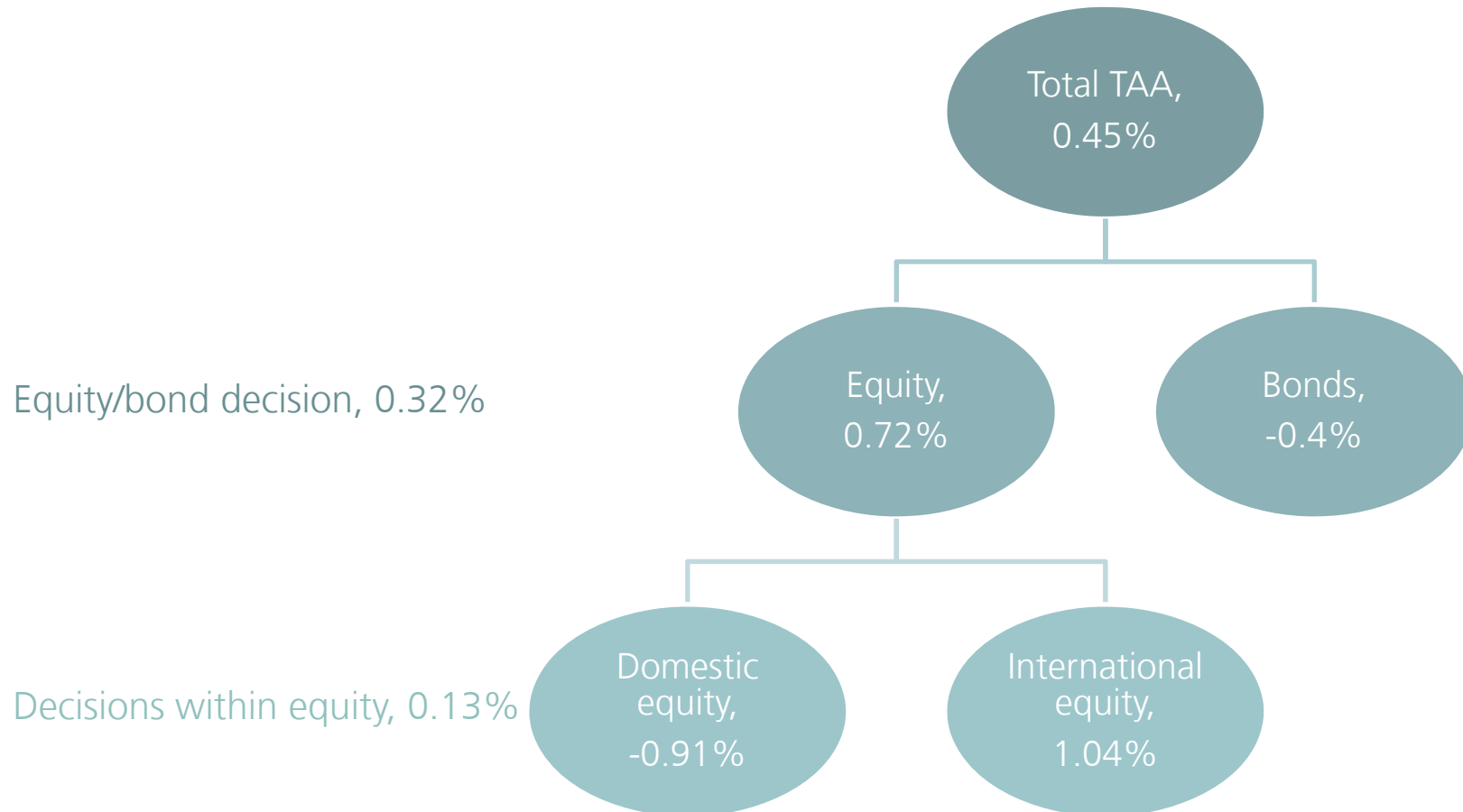


Simplistic approach

Asset	Portfolio weight	Benchmark weight	Active weight	Return	Impact (active weight x return)
Domestic equity	35%	40%	-5%	7%	-0.35%
Int. equity	25%	10%	15%	8%	1.2%
Bonds	40%	50%	-10%	4%	-0.4%
Total	100%	100%	0%		0.45%



Proposed approach



Dividing into major groups

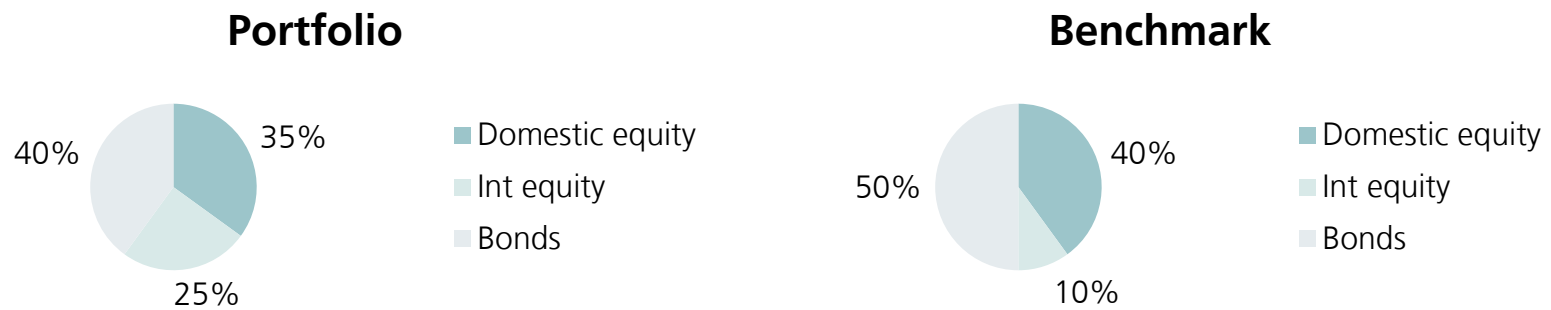
We have two groups at the first level in this example: equity and bonds

Equity group active return (formula (2)) is equal to:

$$\sum_{i \in G_{Equity}} R_i (w_i - w'_i) = 7\% \cdot (35\% - 40\%) + 8\% \cdot (25\% - 10\%)$$

Bond group active return (formula (2)) is equal to:

$$\sum_{i \in G_{Bonds}} R_i (w_i - w'_i) = 4\% \cdot (40\% - 50\%)$$

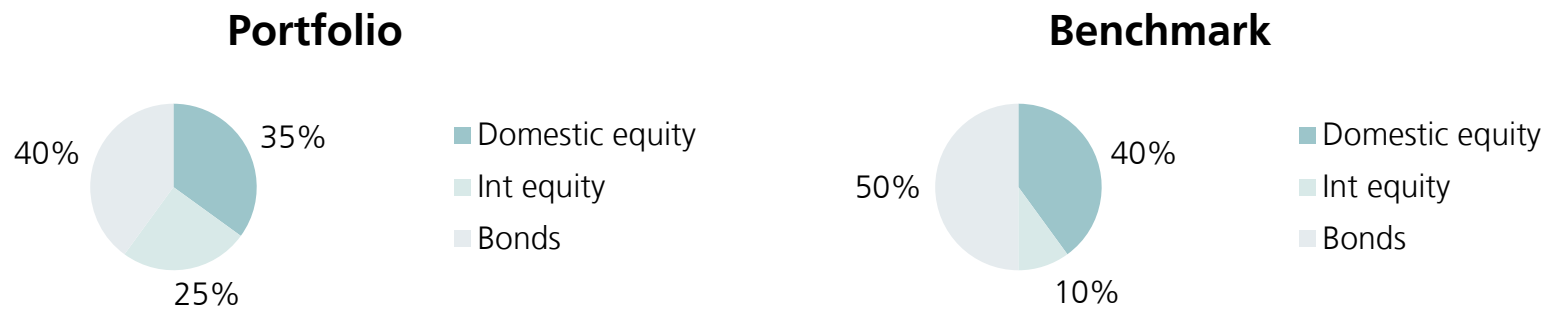


Measuring impact of decisions within groups

For equity, the impact of decisions within the group (formula (3)) is equal to:

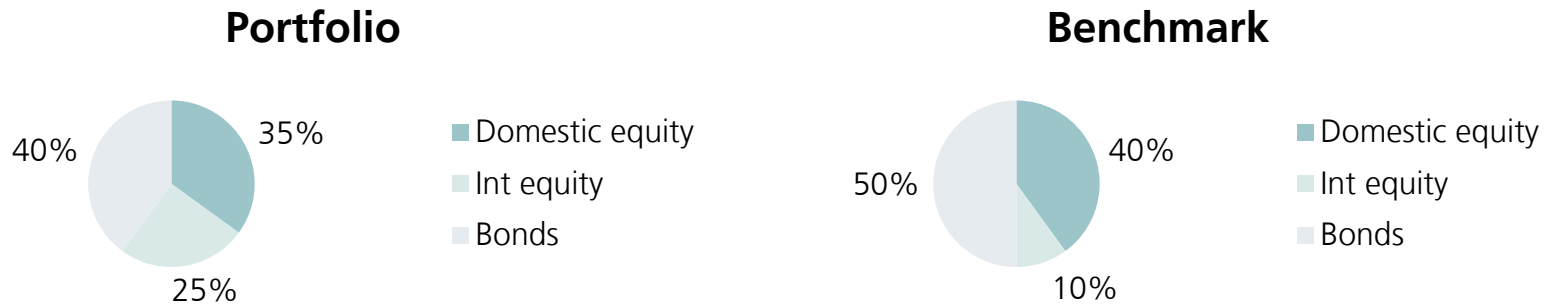
$$\sum_{i \in G_{Equity}} R_i \left(w_i - w'_i \frac{\sum_{i \in G_{Equity}} w_i}{\sum_{i \in G_{Equity}} w'_i} \right) = 7\% \cdot \left(35\% - 40\% \cdot \frac{60\%}{50\%} \right) + 8\% \cdot \left(25\% - 10\% \cdot \frac{60\%}{50\%} \right) =$$

$$= 7\% \cdot (35\% - 48\%) + 8\% \cdot (25\% - 12\%) = -0.91\% + 1.04\% = 0.13\%$$

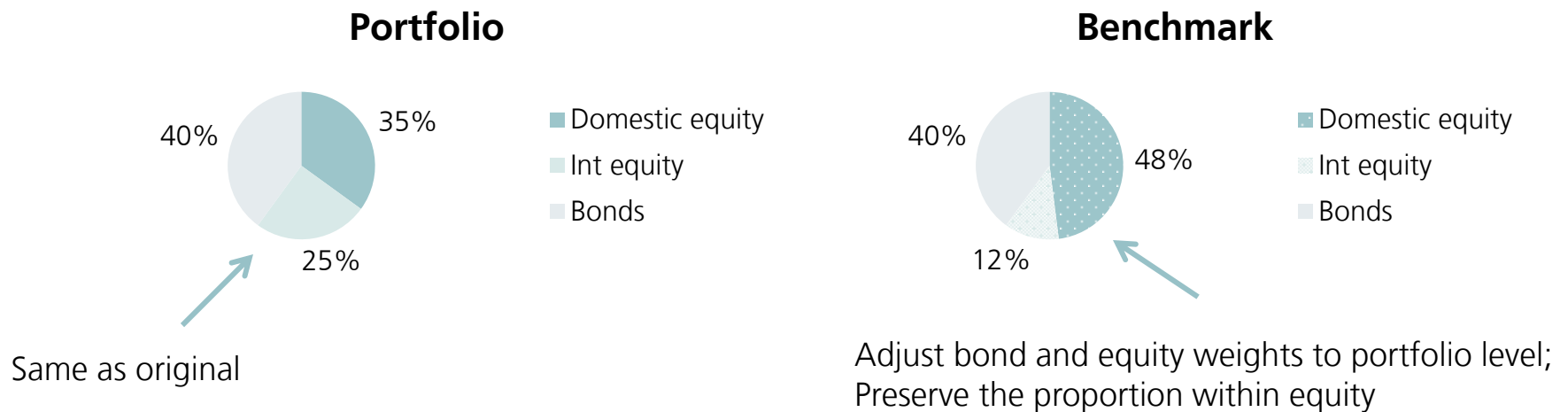


Weights to measure impact of decisions within groups

Original weights



Weights after the benchmark adjustment (formula (3)):



Measuring impact of decisions within groups

For bonds, the impact of decisions within the group (formula (3)) is equal to zero, as there is just one asset in the group:

$$\sum_{i \in G_{Bonds}} R_i \left(w_i - w'_i \frac{\sum_{i \in G_{Bonds}} w_i}{\sum_{i \in G_{Bonds}} w'_i} \right) = 4\% \cdot \left(40\% - 50\% \cdot \frac{40\%}{50\%} \right) = 4\% \cdot (40\% - 40\%) = 0\%$$

Measuring impact of group under/overweight

For equity, the impact of the group under/overweight (formula (4)) is equal to:

$$\begin{aligned} \sum_{i \in G_{Equity}} R_i w'_i \left(\frac{\sum_{i \in G_{Equity}} w_i}{\sum_{i \in G_{Equity}} w'_i} - 1 \right) &= 7\% \cdot 40\% \cdot \left(\frac{60\%}{50\%} - 1 \right) + 8\% \cdot 10\% \cdot \left(\frac{60\%}{50\%} - 1 \right) = \\ &= (7\% \cdot 40\% + 8\% \cdot 10\%) \cdot \left(\frac{60\%}{50\%} - 1 \right) = 7\% \cdot 8\% + 8\% \cdot 2\% = 0.72\% \end{aligned}$$

For bonds, the impact of the group under/overweight (formula (4)) is equal to:

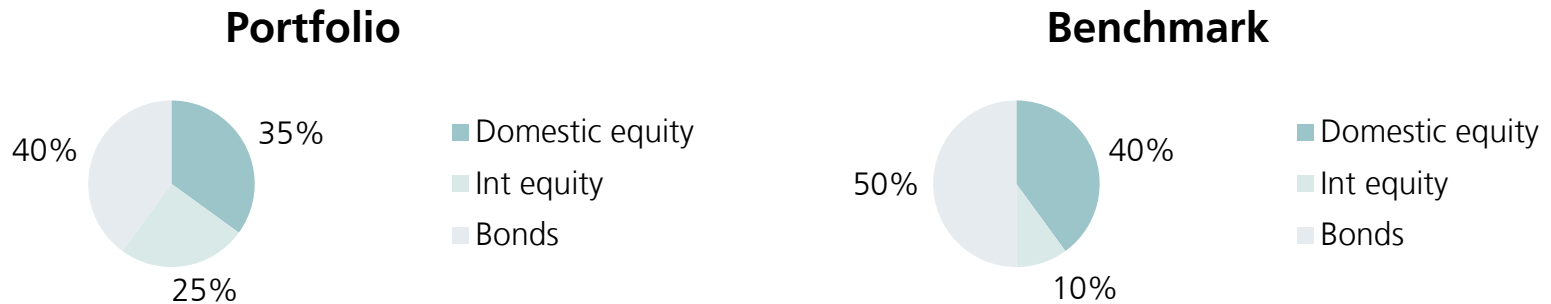
$$\sum_{i \in G_{Bonds}} R_i w'_i \left(\frac{\sum_{i \in G_{Bonds}} w_i}{\sum_{i \in G_{Bonds}} w'_i} - 1 \right) = 4\% \cdot 50\% \cdot \left(\frac{40\%}{50\%} - 1 \right) = 4\% \cdot (-10\%) = -0.4\%$$

The overall impact of the equity/bond decision is equal to:

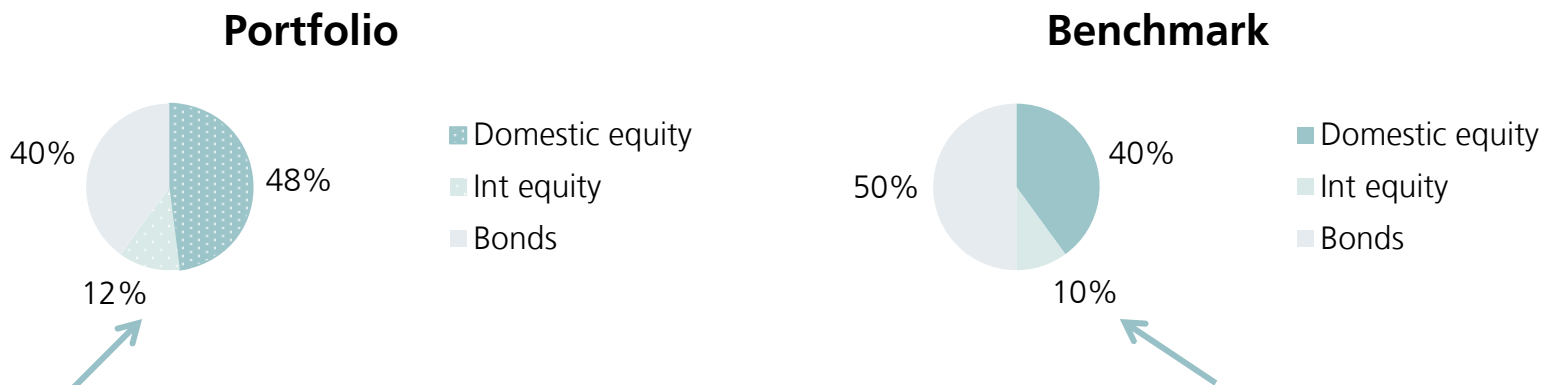
$$0.72\% - 0.4\% = 0.32\%$$

Weights to measure impact of group under/overweight

Original weights



Weights after using benchmark proportions in the portfolio (formula (4)):



Keep bond and overall equity weights as original;
Use the benchmark proportions within equity

Same as original

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